The theoretical ANOVA table can be shown as follows with the mean squares (MS) re-produced from the result from the Genstat outputs

DF e Blocks:Wplots Blocks MS

Between Blocks 5 1 4 12 3175.05556

Between Blocks:Wplots

Variety 2 1 4 0 893.18056

Residual 10 1 4 0 601.33056

Within Blocks.Wplots

Nitrogen 3 1 0 0 6673.5

Variety\*Nitrogen 6 1 0 0 53.625

Residual 45 1 0 0 177.08333

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source of variation | DF | MS | EMS | Variance components |
| Between Blocks | 5 | *S1* |  |  |
| Between Blocks:Wplots |  |  |  |  |
| Variety | 2 |  |  |  |
| Residual | 10 | *S2* |  |  |
| Within Blocks.Wplots |  |  |  |  |
| Nitrogen | 3 |  |  |  |
| Variety\*Nitrogen | 6 |  |  |  |
| Residual | 45 | *S3* |  |  |

The standard error of differences (SED) of the variety is calculated from the residual MS in the Between Whole-plots Within Blocks stratum as

> sqrt(2 \* 601.33056 / 24)

[1] 7.078904

The SED of the nitrogen is calculated from the residual MS in the Within Whole-plots Within Blocks stratum as

> sqrt(2 \* 177.08333 / 18)

[1] 4.435755

The SED of the interaction is calculated from the residual MS in the Within Whole-plots Within Blocks stratum as

> sqrt(2 \* 177.08333 / 6)

[1] 7.682954

This SED is comparing between the means within the same whole-plots, because the residual MS does not contain the variance component of between whole-plots.

Thus, a different SED including the variation of between whole-plots can be constructed by combining the variance components of between whole-plots, , and within whole-plots, .

The can be obtained directly from the residual MS in the Within whole-plots Within Blocks stratum, which is 177.08333. The is calculated from the differences between the residual MS in the Between Whole-plots Within Blocks stratum and the residual MS in the Within Whole-plots Within Blocks stratum which is

> (601.33056 - 177.08333)/4

[1] 106.0618

The SED of the interaction which contains the variation of between and within whole-plots is given by

> sqrt(2 \*(177.08333 + 106.0618)/6)

[1] 9.715025

The next step is to compute the effective degrees of freedom (EDF) associated with each of these three treatment effects. For the variety and nitrogen, the EDF are kept consistent at 10 and 45, respectively, because no extra information can be gain from the other strata.

The EDF of the interaction, without the variation between the whole-plots, is also kept consistent at 45. If the variation between the whole-plots is included, the new EDF need to be approximated based the variance component estimates of the experimental data. For this example, the and are computed as 106.0618 and 177.08333, respectively.

The EDF is then computed from the Satterthwaite formula based on the new MS estimate of .

The numerator of the Satterthwaite formula is given by

and denominator of the Satterthwaite formula

where and denote the residual MS in the in the Between Whole-plots and Within Whole-plots Within Blocks strata as shown in the Table above.

The result of the calculation gives

> nu = (177.08333 + 601.33056/4 - 177.08333 /4)^2

> den = 177.08333 ^2/(45) + (601.33056/4)^2/(10) + (177.08333 /4)^2/(45)

> nu/den

[1] 26.72016

The EDF from this calculation is not the same as the result from Genstat. This may be due one of the coefficients for constructing the new MS estimate being negative.

Instead, the numerator of the Satterthwaite formula is given by

and denominator of the Satterthwaite formula

where and denotes the residual MS in the in the Between Whole-plots and Within Whole-plots Within Blocks strata as shown in the Table above.

The result of the calculation gives

> nu = (3 \* 177.08333/4 + 601.33056/4)^2

> den = (3 \* 177.08333 /4)^2/(45) + (601.33056/4)^2/(10)

> nu/den

[1] 30.23078

The EDF from this calculation is now the same as the result from Genstat. Thus, it is important to have all coefficients are positive.

I then used the method described in the Richard and Kathy’s paper, where the EDF can be approximate as twice the square of the mean divided by the variance. The combination of the variance component estimates, i.e. , is used as the mean.

The variance is computed from the inverse of the Fisher information matrix. The Fisher information matrix is derived from the second derivative of the log-likelihood. The log-likelihood function is obtained based on the assumption that the MS have chi-square distribution which can be written as

where denotes the DF associated MS in and is the linear combination of the variance component estimates.

The Fisher information matrix can then be shown as

Since the EMS are assumed to be the same as the MS,

However, we are interested in each of the variance component estimates; thus, needs to be transformed to , where is a vector containing .

The transformation can be achieved by pre and post-multiply by a matrix, denoted by, containing the coefficients of the variance component in the EMS, i.e.

For this example, the matrix is given by

The variance-covariance matrix is the inverse of and for this example is

The estimated variance of is then given by the sum of four elements in the top left 2 x 2 corner which is 5303.943.

Thus, the EDF can then be derived as

> 2\* (177.08333 + 106.0618)^2 / 5303.943

[1] 30.23078

which confirms the result from Genstat.